AP Calculus BC

Continuity Part 2

1) $x = -1$

I. $f(-1) = 4$

II. $\lim_{x \to -1^-} f(x) = 4$

$\lim_{x \to -1^+} (x+2) = 1$

$\lim_{x \to -1} f(x)$ DNE

$f(x)$ is not cont. @ $x = -1$

$x = 2$

I. $f(2) = 4$

II. $\lim_{x \to 2^-} (x+2) = 4$

$\lim_{x \to 2^+} (2^x) = 4$

$\lim_{x \to 2} f(x) = 4$

III. $f(2) = \lim_{x \to 2} f(x) = 4$

$f(x)$ is cont. @ $x = 2$

2) $f(x) = \begin{cases} x - x^2, & x < 1 \\ x, & x = 1 \\ \ln x, & x > 1 \end{cases}$

I. $f(1) = 1$

II. $\lim_{x \to 1^-} (x - x^2) = 0$

$\lim_{x \to 1^+} (\ln x) = 0$

$\Big\} \lim_{x \to 1} f(x) = 0$

III. $\lim_{x \to 1} f(x) \neq f(1)$

$f(x)$ is not cont @ $x = 1$.

3) $x = \frac{\pi}{2}$

I. $f(\frac{\pi}{2}) = 0$

II. $\lim_{x \to \frac{\pi}{2}^-} f(x) = 0$

$\lim_{x \to \frac{\pi}{2}^+} (\tan x)$ DNE

$f(x)$ is not cont @ $x = \frac{\pi}{2}$

$x = \pi$

I. $f(\pi) = 0$

II. $\lim_{x \to \pi^-} (\tan x) = 0$

$\lim_{x \to \pi^+} \sin x = 0$

$\Big\} \lim_{x \to \pi} f(x) = 0$

III. $f(\pi) = \lim_{x \to \pi} f(x)$

$f(x)$ is cont. @ $x = \pi$

4) $f(x) = \begin{cases} 3-x^2, & x \le 4 \\ x+k, & x > 4 \end{cases}$

I. $f(4) = -13$

$\lim_{x \to 4^-} (3-x^2) = \lim_{x \to 4^+} (x+k)$

$-13 = 4+k$

$\boxed{k = -17}$

5) a) $f(x)$ has an infinite disc @ $x = -1$

b) $g(x) = \dfrac{x+2}{x^2-2x+8} = \dfrac{x+2}{(x-4)(x+2)}$

$g(x)$ has a removable disc @ $x = -2$
& an infinite disc @ $x = 4$.

c) $h(x) = \dfrac{x^2+2x-3}{x+3} = \dfrac{(x+3)(x-1)}{x+3}$

$h(x)$ has a removable disc @ $x = -3$

d) $f(x) = \sec x = \dfrac{1}{\cos x}$ $\cos x = 0$ @ $x = \frac{\pi}{2}, \frac{3\pi}{2}$

$f(x)$ has infinite disc @ $x = \frac{\pi}{2}, \frac{3\pi}{2}$.

e) $f(x) = \begin{cases} x^2+3, & x \le -1 \\ 5x-2, & x > -1 \end{cases}$

$\lim_{x \to -1^-} (x^2+3) = 4$ $f(x)$ has a jump disc @ $x = -1$

$\lim_{x \to -1^+} (5x-2) = -7$

6) C